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Scaling of stiffness in Ising spin glasses

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Abstract. New studies of the $T=0$ scaling theory of Gaussian and bimodal spin glasses, based on exact transfer matrix calculations and on Migdal-Kadanoff scaling, are performed. We focus on the area and length dependencies of the energy sensitivity to changes in the boundary conditions and on the role of vacancies in the bimodal model. We calculate the fractal dimensionality of domain walls in the 2D bimodal systems, valid for short length scales.

1. Introduction

Recently, much progress has been made in the studies of Ising spin glasses (ISG) with short range couplings using a $T=0$ scaling technique [1-5]. This approach is based on the observation that for all $T<T_c$ the behaviour of a system at long length scales should be governed by a $T=0$ fixed point. The conclusion reached in this way has been that the $D=3$ ISG has an equilibrium spin-glass-paramagnet transition at a non-zero T_c whereas the corresponding Heisenberg system does not. These results have been subsequently confirmed by detailed Monte Carlo simulations [6-8]. An Imry-Ma [9] like analysis based on plausible assumptions [3, 10, 11] has led to several new predictions many of which contradict a mean-field analysis of the infinite range model [6]. Among these predictions is the absence of the spin-glass phase in a magnetic field and lack of an infinite number of valleys in the free energy landscape.

The basic concept of the $T=0$ scaling theory is that of a scaling stiffness or a scale-dependent coupling energy, $\delta E(L)$. This coupling is determined by studying the sensitivity to boundary conditions [1-4] of the ground-state energy of finite blocks of length L. $\delta E(L)$ is a characteristic measure of that sensitivity. In the ordered phase at $T=0$

$$\delta E(L) \approx L^y. \tag{1}$$

For systems below the LCD, y is negative and a phase transition occurs at $T=0$. On the other hand, above the LCD y is positive and the transition occurs at a non-zero T_c. Below T_c, the free-energy sensitivity to boundary conditions is given by

$$\delta F(L) \approx Y(T) L^y \tag{2}$$

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where $Y(T)$ vanishes at T_c as a power-law [12] and the exponent y is governed by the $T=0$ fixed point. In the paramagnetic phase $\delta F(L)$ should decay exponentially with the size of the system.

For systems above the LCD the exponent y determines the power-law decay of nonlinear correlations in the ordered phase [3, 10] and the non-analytic dependence of the magnetization on the magnetic field [3]. For systems below the LCD the correlation length diverges as $T \to 0$. The exponent of this divergence is given by $\nu = -1/y$.

In this paper we present the results of new studies of ISGs using the $T=0$ scaling approach and the transfer matrix method to two- and three-dimensional systems. Our studies extend the studies by Bray and Moore [3] and McMillan [4] and are similar to our recent analysis of Potts spin glasses [13]. We consider systems in which the probability distribution of the exchange coupling is either Gaussian (GSG) or bimodal (BSG). In the latter case this distribution is given by $P(J_{ij}) = \frac{1}{2}[\delta(J_{ij}-1) + \delta(J_{ij}+1)]$. We also discuss systems with vacancies in which a specified fraction of the exchange bonds is missing. The presence of vacancies is expected to eliminate the dependence of physical quantities on the parity of L. We consider the scaling behaviour of blocks which are not necessarily cubic. In this way we determine exponents characterizing the length and area dependence of scaling variables like δE. The length, l, here is measured along the direction in which varying boundary conditions are applied. Our notation is explained in greater detail in section 3. Advantages of this geometry are marked in 3D calculations where one can study only few small sizes of systems shaped in cubes. Furthermore, our studies enable us to understand the origin of the D-dependence of the scaling exponents. For instance, the exponent y is found to have a trivial D-dependent component which adds to a D-independent exponent characterizing scaling with l. It is interesting to point out that in the case of Potts spin glasses [13] the scaling with l is, in contrast, D-dependent.

Our other focus is on the role of vacancies, particularly in the case of BSG. It should be pointed out that there has been, in the literature, some controversy regarding the nature of ordering of two-dimensional (2D) bimodal systems: is this system paramagnetic or critical (characterized by an infinite correlation length) at $T=0$? It is possible that an extreme degeneracy of spin configurations may lead to a state unbounded by energy barriers and thus paramagnetic. This in fact happens for a BSG on the Sierpinski gasket [14]. In the case of regular lattices, however, the evidence seems contradictory. On the one hand, an extrapolation of the transfer matrix calculations of the partition function by Morgenstern and Binder [15] to $T=0$ suggested that the 2D BSG is paramagnetic in the ground state. Our own preliminary studies [16] (albeit with small statistics and without a sufficient quenching to the ground state) also pointed to the same conclusion. On the other hand, extensive Monte Carlo calculations by Young [17], McMillan [18], and Swendsen and Wang [19] as well as the $T=0$ transfer matrix analysis of Bray and Moore [3], showed that the $T=0$ state was characterized by an infinite correlation length. In this paper we demonstrate that this system appears to be spin glassy on short-length scales. On large-length scales, we suggest that, there is a crossover to a percolation situation in which zero effective block couplings span the system. This behaviour (naively suggestive of paramagnetism) is also seen in the simple Migdal–Kadanoff rescaling schemes which we discuss in section 2. Unfortunately, as discussed in section 4, this does not unequivocally settle the question of the nature of the $T=0$ phase.

In section 4 we present $T=0$ transfer matrix results on the exponent y in ISGs. In section 5 we discuss the scaling of the fraction of non-zero couplings in 2D BSG. In section 6 we discuss the calculation of an effective fractal dimensionality of the domain wall interface for the bimodal model.

2. Migdal–Kadanoff analysis

The Hamiltonian for the ISG is given by

$$H = -\sum_{\langle ij\rangle}^{N} J_{ij} S_i S_j \tag{3}$$

where $\langle ij\rangle$ denotes a summation over nearest neighbours, the spin S_i takes on values ± 1, and the spins are located on the sites of a D-dimensional cubic lattice.

As a guide it is useful first to map out the properties of an ISG by using the simplest renormalization group scaling scheme, that of Migdal and Kadanoff [20] (MK). In this scheme one determines the effective block coupling on larger and larger length scales by performing the following transformation at each scale. We first reduce b bonds, $K_j = J_j/kT$ $(j = 1, \ldots, b)$, in series into one effective bond K'_{is} by decimating out the spins in the middle which are connected by the bond in series. We then move the b^{D-1} effective bonds in parallel to form a hypercubic lattice with a lattice constant which is b times larger and with the renormalized exchange coupling given by [3, 21, 22]†

$$K' = \sum_{i=1}^{b^{D-1}} K'_{\mathrm{is}} \tag{4a}$$

where

$$K'_{\mathrm{is}} = \ln\left[1 + 2\left(\prod_{j=1} Z_j - 1\right)^{-1}\right] \tag{4b}$$

with

$$Z_j = 1 + 2/[\exp(K_j) - 1]. \tag{4c}$$

The renormalization procedure starts by generating a pool of, typically, 10 000 exchange couplings which describe the system on the microscopic level. The next step is to create a new equally sized pool of rescaled couplings by picking randomly b^D bonds for each new coupling and by transforming the bonds according to the recursion relation (4). The new couplings now form a starting pool in the second-stage rescaling, and so on.

In the $T = 0$ limit (4) reduces to

$$J'_{\mathrm{is}} = \mathrm{sign}(J_1 \ldots J_b) \min(|J_1|, \ldots, |J_b|) \tag{5}$$

provided that none of the couplings is equal to zero. In the case of bimodal distribution it may happen that half of the effective bonds K'_{is} in the parallel arrangement is exactly the negative of the other half resulting in a zero overall coupling K'. In the next stage of the rescaling procedure this zero coupling produces a vanishing serial contribution K'_{is}.

At each length scale we characterize the pool of the couplings by their mean and dispersion, σ. We study these quantities as a function of the number of rescaling stages, n. Both for Gaussian and bimodal couplings the mean stays equal to zero whereas the

† The Migdal–Kadanoff method has been used previously by many other workers to study spin glasses. Our intention here is to summarize the known results and to add a few new results, e.g. for the situation with vacancies, to set the stage for the rest of the paper.

dispersion either grows or decreases, depending on dimensionality and on the choice of the length rescaling factor *b*. As in the discussion of δE, the growth is taken as an indication of spin-glass order, an algebraic decay suggests glassy order only at $T=0$, and an exponential decay indicates paramagnetic behaviour.

Figures 1 and 2 show the flow of σ for the GSG and BSGs. Consider first the case of $D=3$. For both models, σ increases algebraically and the presence of vacancies

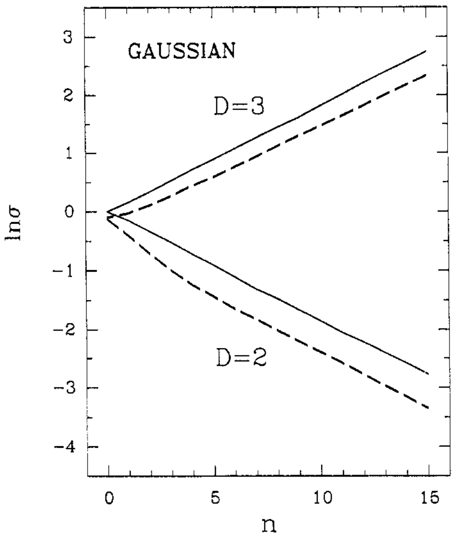

Figure 1. Dependence of the dispersion of the distribution of exchange couplings on the number of iterations in the Migdal–Kadanoff scheme at $T=0$ and for $b=2$. Straight lines on this plot correspond to a power-law dependence on the size of the system. The microscopic level couplings are Gaussian with unit dispersion. The broken lines refer to systems with a vacancy content of 0.2.

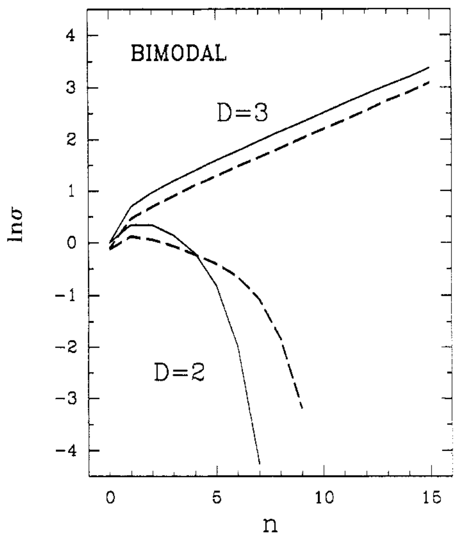

Figure 2. Same as in figure 1 but for the bimodal couplings at the microscopic level.

does not affect the slopes in any significant manner. The slopes on the $\sigma - n$ plane are all of order 0.18 which translates into the exponent y being equal to $0.18/\ln b = 0.26$. This agrees well with

$$y_{\mathrm{BM3}} = 0.19 \tag{6}$$

obtained by Bray and Moore [3] by the transfer matrix method for the $D=3$ systems. For the 2D GSG we get $y = -0.24$, with and without vacancies, which differs by 0.5 from the $D=3$ result and is close to

$$y_{\mathrm{BM2}} = -0.29 \tag{7}$$

calculated by Bray and Moore.

In the 2D bimodal case the situation is very different: the effective coupling in the MK procedure decreases exponentially fast with n. The decay appears to be slowed by the presence of vacancies. The transfer matrix approach, however, predicts a power-law decay. Further discrepancies occur when one compares the scaling behaviour of the fraction of couplings, p, which are non-zero. The MK method for the 2D BSG yields an exponential decrease in p, similar to the behaviour of σ seen in figure 2, whereas Bray and Moore get a power-law, $p \sim L^{-\eta}$, with

$$\eta = 0.2 \tag{8}$$

consistent with the existence of an ordered ground state. For the 3D BSG the transfer matrix method finds no vanishing effective couplings but the simple rescaling suggests a slow (logarithmic or with an exponent 0.03) increase in p to 1.

It is interesting to note that setting $b=3$ yields essentially the same results as scaling with $b=2$ ($y=-0.275$ and 0.245 in 2D and 3D respectively). The only difference appears in the behaviour of the 2D BSG: the $b=3$ MK method suggests that $y=0$ without vacancies and yields an exponential decay of δE with vacancies.

We shall see in sections 3 and 4 that the exponential decays of $\delta E(L)$ and of $p(L)$ in the 2D BSG are also borne out by the transfer matrix method if one fixes the area and varies 1 and are not merely artefacts of the Migdal-Kadanoff method. These phenomena, on the other hand, are not seen when one considers small square samples. This suggests that the small length $T=0$ spin-glass behaviour crosses over, at larger scales, to a situation where the zero couplings begin to percolate.

3. Method of calculation

In order to study the sensitivity to boundary conditions we consider blocks of $A(l+1)$ Ising spins. The parameter A is the transverse area of the sample and l its length in the direction in which differing boundary conditions are applied. For cubic samples $l=L$ and $A=L^{D-1}$.

In the planar $(D-1)$ directions, periodic boundary conditions are applied. In the longitudinal direction, each of the spins in the first and last column ($D=2$) or plane ($D=3$) are fixed randomly in one of the two states. This mimics the influence of neighbouring blocks on the finite block under study. The domain wall is created by turning the spin states on one boundary upside down with the spins on the other boundary held fixed. The difference in the ground-state energies is denoted by ΔE. It can be either positive or negative; we define δE

$$\delta E = \langle |\Delta E| \rangle_c \tag{9}$$

where $\langle\ldots\rangle_c$ denotes the configurational average over samples. Equally well we could consider the root mean square ΔE.

An alternative way to characterize sensitivity to changes in the boundary conditions is to study δE when A is held fixed and l is varied or the other way around. In an ordered phase we should observe

$$\delta E(A, l) \approx l^y f(A/l^{D-1}). \tag{10}$$

In the limit $A \gg l^{D-1} \gg 1$, one expects that the area dependence for δE, for frustrated systems [1], simplifies to $A^{1/2}$. The reason is that, on average, half of the interface spins benefit from a change in the boundary conditions and the other half lose. The energy sensitivity to boundary conditions is a fluctuation effect, and thus of order $A^{1/2}$. It has been demonstrated [23] that this remains a reasonable approximation for squares or cubes. However, one would expect that it would breakdown in the one-dimensional regime $l \ll A \ll l^{D-1}$ with a possibly complicated crossover behaviour. From (10),

$$\delta E \approx A^{1/2} l^x \tag{11}$$

where

$$x = y - (D-1)/2. \tag{12}$$

The exponent x could still depend on D but, at least in the Ising SGs but this does not seem to be the case, as we shall see in section 4.

Our calculations were done using the transfer matrix method as used by Bray and Moore [3] ($T=0$) and Morgenstern and Binder [24] and similar to our studies of the Potts spin glasses [13]. The details can be found in the above references.

4. The $T = 0$ scaling exponent y

Consider first the square and cubic samples of SGs. For each $L \times L$ sized system ($L \leqslant 10$) we took at least 10 000 samples into account in the configurational average. For the cubic samples we considered 4200 samples for $L = 4$ and 10 000 samples for smaller Ls. The transfer matrix results for the GSG with and without vacancies are shown in figure 3. The concentration of vacancies is denoted by c. In the 2D case without vacancies we calculated only the $L = 8$, 9, and 10 data points and took the remaining ones from [3]. We see that the presence of vacancies does not affect the exponents. In $D = 3$ we get

$$y = 0.19 \pm 0.02 = y_{\mathrm{BM3}} \qquad (\mathrm{GSG}, D=3). \tag{13a}$$

For $D = 2$ we combine the $c = 0$ and $c = 0.2$ results and get from the fit

$$y = -0.31 \pm 0.02 \qquad (\mathrm{GSG}, D=2) \tag{13b}$$

which is close to y_{BM2} given by (7). The errors bars have been obtained using the usual statistical measures and are based on the configurational averaging over the distribution of exchange interactions. Systematic errors arising due to small sizes considered are hard to estimate—this remark refers also to all of the data shown from now on. The physical conclusions obtained from (13) are qualitatively the same as obtained from the MK studies.

Consider now the BSG. Figure 4 shows the transfer matrix results obtained with statistics similar to the Gaussian case. The $D = 3$ results are consistent with those obtained for the 3D GSG: the two systems, with and without vacancies, are in the same

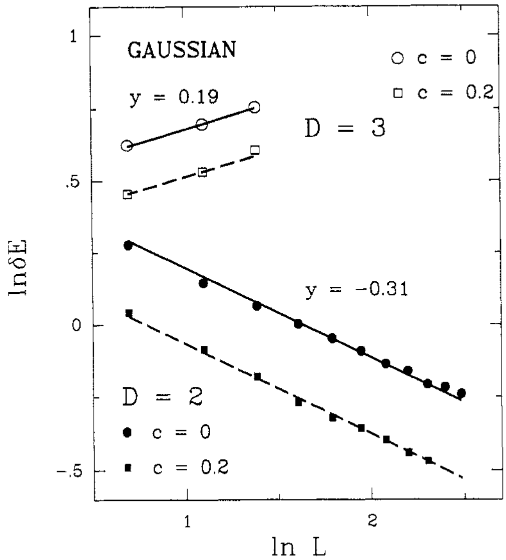

Figure 3. L-dependence of δE for the 2D and 3D GSGs. The results are obtained by the transfer matrix method. The data points denoted by circles (squares) refer to systems without (with) vacancies.

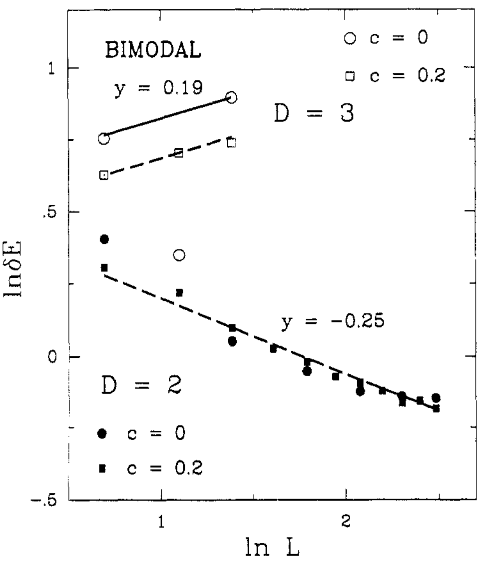

Figure 4. Same as in figure 3 but for the 2D and 3D bimodal spin glasses. One open circle next to the $D = 2$ data points belong actually to the $D = 3$ line. This is an example of a strong odd-even effect.

universality class since the discreteness in the microscopic exchange constants becomes irrelevant in scaling towards an infinite coupling.

In the 3D BSG $c=0$ case, the line joining the $L=2$ and $L=4$ data points does not go through the $L=3$ point. This is due to the 'odd-even' effect which, as noted by Bray and Moore [3], disappears when one introduces vacancies. One encounters this effect already in the case of uniform antiferromagnets. Suppose the periodic boundary conditions are imposed in the planes of such an antiferromagnet. If L is odd then the planes are always frustrated but this does not happen when L is even. Thus in order to calculate the exponent y one should consider odd and even L separately. A similar situation is found in BSGs with $c=0$. Vacancies 'relieve' some of the planar frustration but it is still more reliable to distinguish between odd and even values of L. We shall encounter the odd-even effect in all quantities we study in this paper and adding vacancies always reduces it.

In striking contrast to the MK results, δE of the 2D BSG does not obey an exponential but a power-law behaviour with the exponent

$$y=-0.25\pm0.03 \qquad (\text{BSG}, D=2) \tag{14}$$

which is somewhat higher than y given by (13b). This suggests that the 2D BSG and GSGs belong to different universality classes. The reason for the different behaviour is that in the Gaussian case strictly zero couplings have measure zero but in the 2D bimodal case such couplings have a weight that increases with the size of the system. It should be noted that in the 2D BSG case the value of y depends on a precise definition of δE since the couplings are discrete and the usual scaling limit is not reached. Furthermore, y (as defined above) may not be simply related to the correlation length exponent ν.

We now turn to the discussion of the length and area dependencies of δE. Figure 5 shows the results pertaining to the 2D and 3D GSG systems without vacancies. In these studies we fixed A and varied l between 3 and 20. We used typically 10 000 samples except when $A=4*4$ in which case we took 4200 samples into account. We find that the $A^{1/2}$ dependence is obeyed very accurately so we focus on the l-dependence which is governed by the $\delta E\sim l^x$ law. We obtain $x=-0.81\pm0.02$ for both 2D and 3D systems. Thus the corresponding exponents y are again given by (13) and we conclude that the difference in y between the two dimensionalities is merely due to the trivial area dependence. It should be noted, however, that in $D=1$ GSG, when there is no frustration, it can be shown [3] that then $x=-1$. At sufficiently large ls, for a fixed A in 2D and 3D systems, the exponent x should switch to this 1D value due to a dimensional crossover. However, surprisingly, we saw only weak indications of this happening for lengths l not exceeding 30.

The behaviour of the BSG is more puzzling. This case is shown in figures 6 and 7. The area dependence is again given by the square-root law for small l. The length dependence in 2D, however, starts off algebraically but it quickly curves into an exponential function. This behaviour seems to be inconsistent with the power-law obtained in square samples. Furthermore, the corresponding characteristic lengths in the exponential decay are not intrinsic but appear to depend on A—the larger the A, the longer the length. The presence of vacancies (figure 7) increases the lengths as well but the deviations from a possible power-law are clearly visible. The observed behaviour is suggestive of a complex dimensional crossover to a 1D situation.

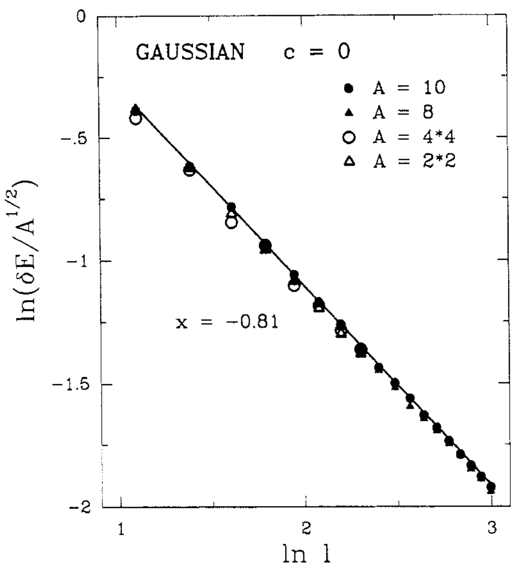

Figure 5. The length dependence of δE divided by $A^{1/2}$ for 2D and 3D Gaussian spin glasses. The values of A are indicated in the figure. Smaller areas were also studied but the corresponding data points essentially lie on the scaling curve.

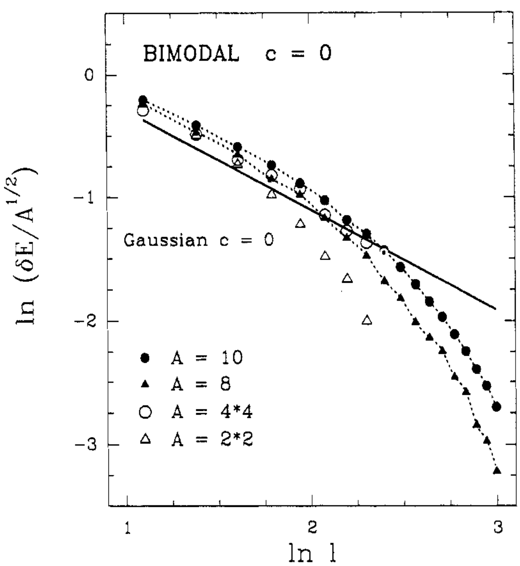

Figure 6. Same as in figure 5 but for the bimodal systems without vacancies. The full line shows, for comparison, the best fit for the Gaussian systems (with $c = 0$).

power-law. Only in the case of $A = 2 * 2$ and $c = 0$ are the deviations due to smallness of A seen. For $A = 3 * 3$ the odd-even effect causes a saturation of δE at a constant value (not shown) but again this behaviour disappears when vacancies are introduced. In the 2D case the initial power-law could be consistent with the exponent $x = -0.75$, which yields $y = -0.25$. This incipient ordering, however, persists on short length scales only.

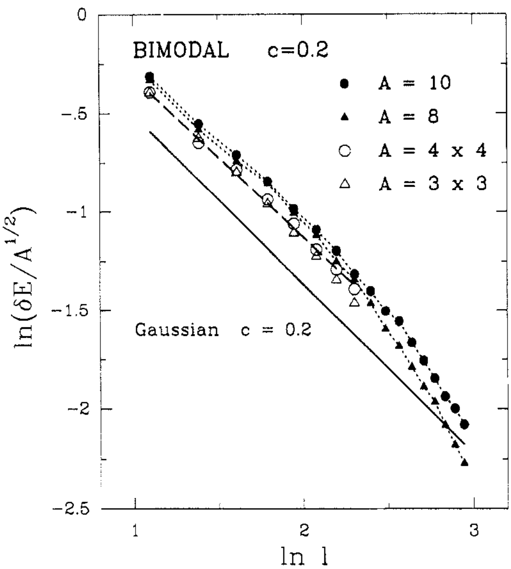

Figure 7. Same as in figure 6 but for the bimodal system with vacancies.

The situation in the 3D BSGs is strikingly different from that in two dimensions [25]. In the 3D case there is a non-zero percolation threshold for vacancies to destroy the order. Stein *et al* [26] have studied ground-state configurations of BSGs in which a fraction x of nearest-neighbour couplings are antiferromagnetic and the rest are ferromagnetic.

5. Scaling of the fraction of non-zero couplings

Bray and Moore [3] have studied the probability, $p(L)$, of finding a non-zero δE in BSGs and found that it is 1 in $D=3$ and in $D=2$ there is a power law decrease at $T=0$

$$p(L) \approx L^{-\eta} \tag{15}$$

with $\eta = 0.20 \pm 0.02$. This exponent describes the algebraic decay of the spin–spin correlations at $T=0$. Morgenstern and Binder [24] obtained 0.4 ± 0.1 for η and McMillan [18] got 0.28 ± 0.04. The 2D GSG has a unique ground state and hence $\eta = 0$ in this case.

Our own results, based on 10 000 square samples are consistent with the power-law obtained by Bray and Moore. We note, however, that, e.g., for $L = 10$ the probability of finding a zero coupling is equal to 0.784. In the scaling picture, we might consider building up a large system by putting together blocks of size $10 * 10$. Since 78% of such blocks have a zero sensitivity to changes in boundary conditions, it seems reasonable to expect that those zero couplings will percolate leading to an overall zero sensitivity of the large system. Such a scenario might lead to deviations from the algebraic decay law for $p(L)$ at sufficiently large L ($L \gg 10$) even for square lattices of edge L. The complex crossover behaviour alluded to in the previous section could thus arise due to a one-dimensional crossover or due to a percolative crossover.

Even assuming that the above picture is realized and that for large enough L, $p(L) \approx 0$, what might one conclude regarding the nature of the $T=0$ phase? Is it paramagnetic or critical? While one may be tempted to connect the decoupling of block spins at long length scales to paramagnetism, such an assumption does not necessarily hold in all cases. An interesting counterexample is that of an antiferromagnetic 3-state Potts model on a square lattice. The highly degenerate ground state of this model leads to zero sensitivity to changes in boundary conditions. An exact calculation by Baxter [27] shows, however, that the $T=0$ state is critical. A similar behaviour for the $D=2$ BSG cannot be ruled out.

6. Fractal dimensionality of interfaces

Ising spin glasses have been found to be chaotic [10, 11, 20, 28]† in the sense that the spin order is sensitive to a temperature change δT at length scales L^* of order $(Y/\sigma\delta T)^{1/\zeta}$, where Y and σ are T-dependent amplitudes associated with the interfacial free energy and entropy, respectively. $\zeta = d_S/2 - y$ is the Lyapunov exponent characterizing the chaotic behaviour and d_S is the fractal dimension of the interface.

In order to determine d_S one has to calculate the mean interface length and see how it scales with L. An easy way [28] to do this, but only in Gaussian systems, is to calculate

$$I_l = \lim_{\lambda \to 0} [\langle(\Delta E - \Delta E(\lambda))^2\rangle_c]/\lambda^2 \tag{16}$$

where $\Delta E(\lambda)$ denotes ΔE in the presence of perturbation

$$J_{ij} \to J_{ij} + \lambda x_{ij} \tag{17}$$

with $\lambda \ll 1$ and x_{ij} drawn from a Gaussian pool of couplings with zero mean and unit dispersion. For a particular sample $\Delta E - \Delta E(\lambda) = (i_l)^{1/2}\lambda z$, where i_l is the actual interface length and z is a normally distributed variable of unit dispersion. Thus $\langle i_l \rangle_c = I_l$. The exponent d_S is defined by

$$I_l \approx L^{d_S}. \tag{18}$$

Figure 8 shows the L-dependence of the effective interface, (16), as determined by the transfer matrix method for the 2D GSG. The statistics are: 10 000 samples for $L \leqslant 8$, 2800 for $L=9$, and 2000 for $L=10$. We get

$$d_S = 1.27 \pm 0.06 \qquad (\text{GSG}, D=2) \tag{19}$$

which agrees with 1.26 obtained by Bray and Moore [28]. Since d_S is larger than $2y$, the Lyapunov exponent ζ is positive which corresponds to chaotic behaviour.

The method of establishing the interface length, as outlined above, is valid only for the Gaussian systems. In the bimodal case we adopt the following procedure. We choose first one kind of boundary condition and find the ground-state energy in the presence of an infinitesimal noise in the couplings. We subtract this energy from the nearest integer value to extract the component which is entirely due to the noise. In the second stage of the procedure we invert spins on the boundary and repeat calculations of the ground-state energy in the presence of exactly the same noise and

† Note that chaotic behaviour was first observed in a hierarchical Ising model with competing interactions by McKay *et al.* The origins of the chaos seem different in the two cases.

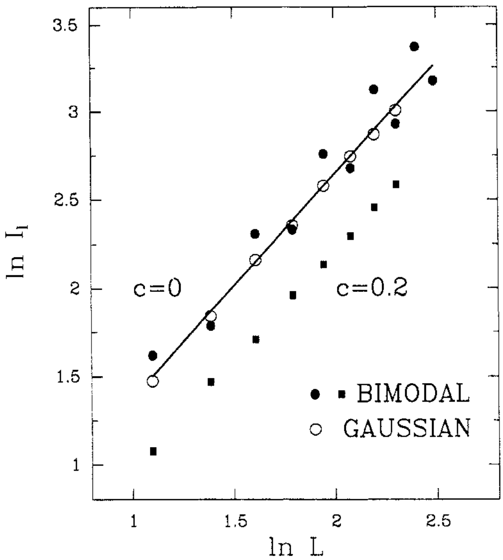

Figure 8. The scaling of the domain wall length for bimodal and Gaussian 2D square systems.

we again extract the noise contribution. The difference of the noise contributions is proportional to the interface length. The procedure is then repeated over the ensemble of couplings (5000 samples) and random noises. The average root mean square difference in the noise contributions is divided by the dispersion of the noise and then plotted in figure 8 for the 2D case.

Figure 8 shows that the fractal dimensionality of the interface in 2D bimodal systems is equal, or at least very close, to the one obtained for the GSG. Again, without vacancies

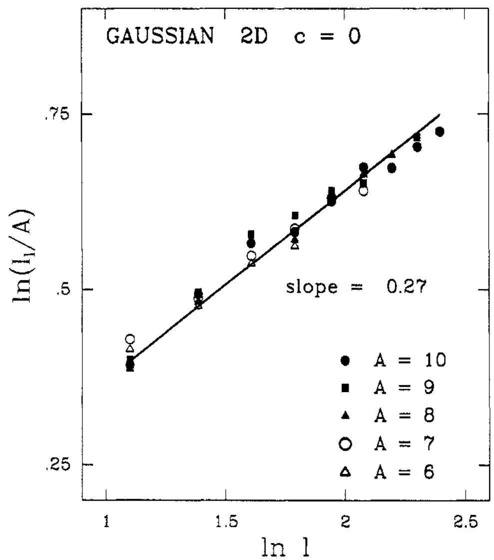

Figure 9. The length dependence of the interface length for the 2D Gaussian spin glasses without vacancies. The area dependence is linear.

the odd-even effect is quite pronounced but the presence of vacancies does not affect d_S. It should be remembered, however, that d_S characterizes the behaviour of the 2D BSG interface only on short-length scales.

Figure 9 shows the l-dependence of the interface for 2D GSG. The area is fixed at values between 6 and 10. The data are consistent with the slope of 0.27 which suggests a linear A-dependence. This trivial A-dependence is consistent with the effective coupling δE scaling as the square root of A.

The A-dependence in $D=2$, combined with x being independent of D suggests that $d_S=2.27$ in the 3D ISG. This in turn would imply that the Lyapunov exponent ζ should be D-independent because both $d_S/2$ and y change by 0.5 between D and $D+1$. Such a D-independent ζ is also found in the Migdal-Kadanoff approximation since $d_S=D$ in that case.

6. Concluding remarks

In summary, we conclude that studying of the scaling stiffness and related quantities as functions independently of l and A brings in new insights and offers numerical advantages. Among these insights is the realization that the $A^{1/2}$ law for the scaling of δE is connected to the linear A-dependence of the length of the fractal domain wall. It is possible that there exists a relationship between the exponents for the l-dependence of the two quantities. It remains an open question whether the physics of the 2D bimodal spin glass is governed by a crossover between the spin-glass behaviour characterizing short length scales and the percolation of soft couplings or 1D behaviour at large length scales leading to paramagnetism.

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